Dynamics of Quantum State in One-Dimensional Open Systems with Random Perturbation: Phase-Space Approach

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Abstract

The subject of this doctoral dissertation is the dynamical analysis of open quantum system in the presence of random potential perturbations using the phase-space formalism. Using the Stratonovich-Weyl quantisation scheme for a one-dimensional system interacting with an N-dimensional environment, the equation of motion for the Wigner function was derived from a microscopic perspective. Due to the adopted assumptions, the results obtained refer to weakly interacting memoryless systems. The resulting equation of motion, after assuming a linear expression for the Lindblad symbol $L(x,p) = ax + bp$, $a, b \in \mathbb{C}$, was expressed in operator form, and subsequently, the second-order split-operator method was applied. The time evolution generator was expressed as the sum of four operators, for which a commutation table was prepared. As a result of the calculations, it was observed that the part of the generator not containing the potential term can be separated exactly, thereby reducing the number of applied Fourier transforms. Next, the equation of motion for the approximated averaged Wigner function of the statistical ensemble was derived, yielding an equation with a diffusion coefficient that is linearly dependent on time, the perturbation strength, and the second derivative of the potential. As model systems, a single Gaussian barrier and a chain of such barriers were considered, and each of these systems was then perturbed by a random vector whose components were independent random variables. For such prepared potentials, the equation of motion for the Wigner function was solved numerically using the split-operator method, with a common Gaussian initial condition. In the C++ implementation of the algorithm, the Fast Fourier Transform (FFTW3) was employed. The dynamical analysis of the studied cases was based on the following measures: statistical - averages, standard deviations, and covariance; phase-space - nonclassicality parameter and Wigner-Shannon entropy; state - purity and Loschmidt echo; distance - the $L^2(\mathbb{R}^2)$-normalised distance between the solution for the unperturbed potential and the approximate solution. These measures were used to study the influence of the environment on the dynamics of the open system. Additionally, the existence of stationary states for the studied cases was analysed.

Keywords: open systems, Lindblad equation, phase-space representation, Wigner function, split-operator method, disordered systems

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