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Summary of doctoral dissertation

"Dynamics of quantum states in phase-space"

The main topic covered in the doctoral thesis is the dynamics of quantum states within phase-space formulation of quantum mechanics in the case of isolatated systems. The primary object that describes the state of the system in phase space is a Wigner function. The time evolution of this function is given by the Moyal equation, for which the classical limit is consistent with the structure of the Liouville equation known from statistical mechanics.

Just as in the case of the Schrödinger equation, the generator in the Moyal equation is a sum of two non-commuting self-adjoint operators related to kinetic and potential parts of the Hamiltonian, and its solution is given by the time evolution operator belonging to the one-parameter strongly continuous unitary group. Due to the fact that in general case it is very difficult to calculate the action of such operator on the Wigner function, *exponential operator splitting* method is used, which relies on approximation of the exact time evolution operator by a product of a few unitary operators where each of them is generated by one operator related to either kinetic or potential part of the Hamiltonian. Action of such type of operators on the Wigner function can be obtained numerically in a very efficient way.

In chapter 4, which is devoted to the most important mathematical results, I present and prove a theorem on the upper bound for local error in Hilbert space norm for *exponential operator splitting* method in the case of abstract evolution equation. Then I present an application of this theorem to the Moyal equation in Hilbert space $L^2(R^2)$. The culmination of these considerations is a formulation and proof of the theorem on the upper bound of global error, which is the final error after completing larger number of time steps according to the *exponential operator splitting* method, in the case of the Moyal equation.

In chapter 5 I discuss the most important physical results concerning time evolution of quantum states obtained via numerical algorithm with precision estimated in the previous chapter. In the first part, I consider bounded systems consisting of various potential wells, and the initial state assumed as a coherent state which is a Gaussian state that saturates Heisenberg uncertainity principle. In the second part I consider a scattering system consisting of a smooth potential barrier and the initial state is assumed as a defected Schrödinger cat state. The Schrödinger cat state is the quantum state that is a superposition of two spatially separated coherent states, and defect is related to reduced amplitude of interference term in the Wigner function which results in decreased purity of that state.

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